INVENTION: Complex Walsh Codes for CDMA

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TECHNICAL FIELD

The present invention relates to CDMA (Code Division Multiple Access) cellular telephone and wireless data communications with data rates up to multiple T1 (1.544 Mbps) and higher (>100 Mbps), and to optical CDMA with data rates in the Gbps and higher ranges. Applications are mobile, point-to-point and satellite communication networks. More specifically the present invention relates to novel complex and hybrid complex Walsh codes developed to replace current real Walsh orthogonal CDMA channelization codes.

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ABSTRACT

The present invention describes a new set of complex Walsh and hybrid complex Walsh orthogonal codes for CDMA channelization encoding and decoding. Current art uses real 2-phase Walsh orthogonal codes for spreading and orthogonal channelization encoding of the data for the CDMA signal. Complex CDMA is widely known to be better than real CDMA and the current art generates complex CDMA by using real Walsh codes together with

pseudo-noise bi-phase (PN) codes to generate a complex CDMA signal. The new 4-phase complex Walsh orthogonal CDMA codes provide fundamental performance improvements which include an increase in the carrier-to-noise ratio (CNR) for data symbol recovery in the receiver, lower correlation side-lobes under timing offsets both with and without PN spreading, lower levels of harmonic interference caused by non-linear amplification of multi-carrier CDMA signals, and reduced phase tracking jitter for code tracking to support both acquisition and synchronization. Hybrid complex Walsh orthogonal CDMA codes increase the choices for the code length by allowing the combined use of complex Walsh and discrete Fourier transform complex orthogonal codes using a Kronecker construction, direct sum constructioin, as well as the possibility for more general functional combining.

BACKGROUND ART

Current art is represented by the recent work on multiple access for broadband wireless communications, the G3 (third generation CDMA) proposed standard candidates, the current IS-95 CDMA standard, the early Qualcomm patents, and the real Walsh technology. These are documented in references 1,2,3,4,5,6. Reference 1 is an issue of the IEEE communications magazine devoted to multiple access communications for broadband wireless networks, reference 2 is an issue on IEEE personal communications devoted to the third generation (3G) mobile systems in Europe, reference 3 is the IS-95 standard primarily developed by Qualcomm, references 4 and 5 are Qualcomm patents addressing the use of real Walsh orthogonal CDMA codes, and reference 6 is the widely used reference on real Walsh technology.

Current art using real Walsh orthogonal CDMA channelization codes is represented by the scenario described in the following with the aid of equations (1) and FIG 1,2,3,4. This scenario considers CDMA communications spread over a common

frequency band for each of the communication channels. These CDMA communications channels for each of the users are defined by assigning a unique Walsh orthogonal spreading codes to each user. The Walsh code for each user spreads the user data symbols over the common frequency band. These Walsh encoded user signals are summed and re-spread over the same frequency band by one or more PN codes, to generate the CDMA communications signal which is modulated and transmitted. The communications link consists of a transmitter, propagation path, and receiver, as well as interfaces and control.

It is assumed that the communication link is in the communications mode with all of the users communicating at the same symbol rate and the synchronization is sufficiently accurate and robust to support this communications mode. In addition, the possible power differences between the users is assumed to be incorporated in the data symbol amplitudes prior to the CDMA encoding in the CDMA transmitter, and the power is uniformly spread over the wideband by proper selection of the CDMA pulse waveform. It is self evident to anyone skilled in the CDMA communications art that these communications mode assumptions are both reasonable and representative of the current CDMA art and do not limit the applicability of this invention.

Transmitter equations (1) describe a representative real Walsh CDMA encoding for the transmitter in FIG. 1. It is assumed that there are N Walsh code vectors W(u) each of length N chips **1**. The code vector is presented by a 1xN N-chip row vector W(u)=[W(u,1),…,W(u,N)] where W(u,n) is chip n of code u. The code vectors are the row vectors of the Walsh matrix W. Walsh code chip n of code vector u has the possible values W(u,n)= +/-1. Each user is assigned a unique Walsh code which allows the code vectors to be designated by the user symbols u=0,1,…,N-1 for N Walsh codes. User data symbols **2** are the set of complex symbols {Z(u), u=0,,1,…,N-1) and the set of real symbols (R(u_R), I(U_I), u_R, u_I =0,1,…,N-1} where Z is a complex symbol and R,I are real symbols assigned to the real, imaginary

communications axis. Examples of complex user symbols are QPSK and OQPSK encoded data corresponding to 4-phase and offset 4-phase symbol coding. Examples of real user symbols are PSK and DPSK encoded data corresponding to 2-phase and differential 2-phase symbol coding. Although not considered in this example, it is possible to use combinations of both complex and real data symbols.

Current real Walsh CDMA encoding for transmitter (1)

1 Walsh codes

W = Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [W(u)] matrix of row vectors W(u)

= [W(u,n)] matrix of elements W(u,n)

W(u) = Walsh code vector u for u=0,1,…,N-1

= [W(u,0), W(u,1), …, W(u,N-1)]

= 1xN row vector of chips W(u,0),….,W(u,N-1)

W(u,n) = Walsh code u chip n

= +/+1 possible values

2 Data symbols

Z(u) = Complex data symbol for user u

$R(u_R)$ = Real data symbol for user u_R assigned to the real axis of the CDMA signal

$I(u_I)$ = Real data symbol for user u_I assigned to the imaginary axis of the CDMA signal

3 Walsh encoded data

Complex data symbols

Z(u,n) = Z(u) sgn{ W(u,n)}

= User u chip n Walsh encoded complex data

Real data symbols

$R(u_R,n)$ = $R(u_R)$ sgn{ $W(u_R,n)$ }

= User u_R chip n Walsh encoded real data

$I(u_I,n)$ = $R(u_R)$ sgn{ $W(u_R,n)$ }

= User u_I chip n Walsh encoded real data

where sgn{ (o) } = Algeraic sign of "(o)"

4 PN scrambling

$P_{R2}(n)$, $P_R(n)$ = Chip n of PN codes for real axis

$P_{I2}(n)$, $P_I(n)$ = Chip n of PN codes for imaginary axis

Complex data symbols:

Z(n) = PN scrambled real Walsh encoded data chips after summing over the users

$$= \sum_u Z(u,n)[\mathrm{P_R(n)} + \mathrm{j\,P_I(n)}]$$

$$= \sum_u Z(u,n)[\mathrm{sgn\{P_R(n)\}} + \mathrm{j\,sgn\{P_I(n)\}}]$$

= Real Walsh CDMA encoded complex chips

Real data symbols:

Z(n) =

$$[\sum_{u_R} R(u_R,n)\,\mathrm{sgn}(P_{R2}(n)) + \mathrm{j}\sum_{u_I} I(u_I,n)\ \mathrm{sgn}(P_{I2}(n))]\ [\mathrm{sgn}\{P_R(n)\} + \mathrm{j\,sgn\{P_I(n)\}}]$$

= Real Walsh CDMA encoded real chips

User data is encoded by the Walsh CDMA codes **3.** Each of the user symbols $Z(u), R(u_R), I(u_I)$ is assigned a unique Walsh code $W(u), W(u_R), W(u_I)$. Walsh encoding of each user data symbol generates an N-chip sequence with each chip in the sequence consisting of the user data symbol with the sign of the corresponding Walsh code chip, which means each chip = [Data symbol] x [Sign of Walsh chip].

The Walsh encoded data symbols are summed and encoded with PN codes **4.** These PN codes are 2-phase with each chip equal to +/-1 which means PN encoding consists of sign changes with each sign change corresponding to the sign of the PN chip. Encoding with PN means each chip of the summed Walsh encoded data symbols has a sign change when the corresponding PN chip is -1, and remains unchanged for +1 values. This operation is described by a multiplication of each chip of the summed Walsh encoded data symbols with the sign of the PN chip. Purpose of the PN encoding for complex data symbols is to provide scrambling of the summed Walsh encoded data symbols as well as isolation between groups of users. Purpose of the separate PN encoding for the real and

imaginary axes is to provide approximate orthogonality between the real and imaginary axes, since the same Walsh orthogonal codes are being used for these axes. Another PN encoding can be used as illustrated in these equations for the combined real and imaginary CDMA signals to provide scrambline and isolation between groups of users.

Receiver equations (2) describe a representative real Walsh CDMA decoding for the receiver in FIG. 3. The receiver front end **5** provides estimates $\{\hat{Z}(n)=\hat{R}(n)+\mathrm{j}\ \hat{I}(n)\}$ of the transmitted real Walsh CDMA encoded chips {Z(n)=R(n)+jI(n)} for the complex and real data symbols. Orthogonality property **6** is expressed as a matrix product of the real Walsh code chips or equivalently as a matrix produce of the Walsh code chip numerical signs. The 2-phase PN codes **7** have the useful decoding property that the square of each code chip is unity which is equivalent to observing that the square of each code chip numerical sign is unity. Decoding algorithms **8** perform the inverse of the signal processing for the encoding in equations (1) to recover estimates $\{\hat{Z}(u)\}$ or $\{\hat{R}(u_R),\ \hat{I}(u_I)\}$ of the transmitter user symbols {Z(u)} or $\{R(u_R),\ I(u_I)\}$ for the respective complex or real data symbols.

Current real Walsh CDMA decoding for receiver (2)

5 Receiver front end provides estimates $\{\hat{Z}(n) = \hat{R}(n)+j\hat{I}(n)\}$ of the encoded transmitter chip symbols {Z(n)=R(n)+jI(n)} for the complex and real data symbols

6 Orthogonality property of real Walsh NxN matrix W

$$\sum_n W(\hat{u},n)\,W(n,u) = \sum_n \mathrm{sgn}\{W(\hat{u},n)\}\,\mathrm{sgn}\{W(n,u)$$

$$= \mathbf{N}\ \delta(\hat{u},\ u)$$

where $\delta(\hat{u},\ u)$ = Delta function of $\hat{u}$ and u

$= 1$ for $\hat{u} = u$

$= 0$ otherwise

7 PN decoding property

P(n)P(n) = sgn{P(n) sgn{P(n)}

= 1

8 Decoding algorithm

Complex data symbols

$$\hat{Z}(u) = N^{-1}\sum_n \hat{Z}(n)\,[\mathrm{sgn}\{P_R(n)\} - j\,\mathrm{sgn}\{P_I(n)\}]\ \mathrm{sgn}\{W(n,u)\}$$

= Receiver estimate of the transmitted complex data symbol Z(u)

Real data symbols

$$\hat{R}(u_R) =$$

$$\mathrm{Real}[\ N^{-1}\sum_n \hat{Z}(n)[\mathrm{sgn}\{P_R(n)\} - j\,\mathrm{sgn}\{P_I(n)\}]\ \mathrm{sgn}\{P_{R2}(n)\}\mathrm{sgn}\{W(n,u_R)\}\ ]$$

= Receiver estimate of the transmitted complex data symbol $R(u_R)$

$$\hat{I}(u_I) =$$

$$\mathrm{Imag}[\ N^{-1}\sum_n \hat{Z}(n)[\mathrm{sgn}\{P_R(n)\} - j\,\mathrm{sgn}\{P_I(n)\}]\ \mathrm{sgn}\{P_{I2}(n)\}\mathrm{sgn}\{W(n,u_I)\}\ ]$$

= Receiver estimate of the transmitted complex data symbol $I(u_I)$

FIG. 1 CDMA transmitter block diagram is representative of a current CDMA transmitter which includes an implementation of the current real Walsh CDMA channelization encoding in equations (1). This block diagram becomes a representative implementation of the CDMA transmitter which implements the new complex Walsh CDMA encoding when the current real Walsh CDMA encoding **13** is replaced by the new complex Walsh CDMA encoding of this invention. Signal processing starts with the stream of user input data words **9.** Frame processor **10** accepts these data words and performs the encoding and frame formatting, and passes the outputs to the symbol encoder **11** which encodes the frame symbols into amplitude and phase coded symbols **12** which could be complex {Z(u)} or real {$R(u_R)$, $I(u_I)$) depending on the application. These symbols **12** are the inputs to the current real Walsh CDMA encoding in equations (1). Inputs {Z(u)}, {$R(u_R)$, $I(u_I)$) **12** are real Walsh encoded, summed over the users, and

scrambled by PN in the current real Walsh CDMA encoder **13** to generate the complex output chips {Z(n)} **14.** This encoding **13** is a representative implementation of equations (1). These output chips Z(n) are waveform modulated **15** to generate the analog complex signal z(t) which is single sideband upconverted, amplified, and transmitted (Tx) by the analog front end of the transmitter **15** as the real waveform v(t) **16** at the carrier frequency f_0 whose amplitude is the real part of the complex envelope of the baseband waveform z(t) multiplied by the carrier frequency and the phase angle ϕ accounts for the phase change from the baseband signal to the transmitted signal.

It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 1 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

FIG. 2 real Walsh CDMA encoding is a representative implementation of the real Walsh CDMA encoding **13** in FIG. 1 and in equations (1). Inputs are the user data symbols which could be complex {Z(u)} or real $\{R(u_R) + jI(u_I)\}$ **17.** For complex and real data symbols the encoding of each user by the corresponding Walsh code is described in **18** by the implementation of transferring the sign of each Walsh code chip to the user data symbol followed by a 1-to-N expander $1\uparrow N$ of each data symbol into an N chip sequence using the sign transfer of the Walsh chips.

For complex data symbols {Z(u)} the sign-expander operation **18** generates the N-chip sequence Z(u,n) = Z(u) sgn{W(u,n)} = Z(u) W(u,n) for n=0,1,…,N-1 for each user u=0,1,…,N-1. This Walsh encoding serves to spread each user data symbol into an orthogonally encoded chip sequence which is spread over the CDMA communications frequency band. The Walsh encoded chip sequences for each of the user data symbols are summed over the users **19** followed by PN encoding with the scrambling sequence

$[P_R(n)+jP_I(n)]$ **21.** PN encoding is implemented by transferring the sign of each PN chip to the summed chip of the Walsh encoded data symbols. Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **22.** The switch **20** selects the appropriate signal processing path for the complex and real data symbols.

For real data symbols $\{R(u_R)+jI(u_I)\}$ the real and imaginary communications axis data symbols are separately Walsh encoded **18,** summed **19,** and then PN encoded **19** to provide orthogonality between the channels along the real and imaginary communications axes. Output is complex combined **19** and PN encoded with the scrambling sequence $[P_R(n)+ jP_I(n)]$ **21.** Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **22.**

It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 2 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

FIG. 3 CDMA receiver block diagram is representative of a current CDMA receiver which includes an implementation of the current real Walsh CDMA decoding in equations (2). This block diagram becomes a representative implementation of the CDMA receiver which implements the new complex Walsh CDMA decoding when the current real Walsh CDMA decoding **27** is replaced by the new complex Walsh CDMA decoding of this invention. FIG. 3 signal processing starts with the user transmitted wavefronts incident at the receiver antenna **23** for the n_u users $u = 1,\ldots, n_u \leq N_c$. These wavefronts are combined by addition in the antenna to form the receive (Rx) signal $\hat{v}(t)$ at the antenna output **23** where $\hat{v}(t)$ is an estimate of the transmitted signal v(t) **16** in FIG. 1, that is received with errors in time Δt, frequency Δf, phase $\Delta\theta$, and with an estimate $\hat{z}(t)$ of the transmitted complex baseband signal z(t) **16** in FIG. 1. This received signal $\hat{v}(t)$ is

amplified and downconverted by the analog front end **24** and then synchronized and analog-to-digital (ADC) converted **25.** Outputs from the ADC are filtered and chip detected **26** by the fullband chip detector, to recover estimates $\{\hat{Z}(n) = \hat{R}(n) + j\,\hat{I}(n)\}$ **28** of the transmitted signal which is the stream of complex CDMA encoded chips $\{Z(n)=R(n)+jI(n)\}$ **14** in FIG. 1 for both complex and real data symbols. The CDMA decoder **27** implements the algorithms in equations (2) by stripping off the PN code(s) and decoding the received CDMA real Walsh orthogonally encoded chips to recover estimates $\{\hat{Z}(u) = \hat{R}(u_R)+j\,\hat{I}(u_I)\}$ **29** of the transmitted user data symbols $\{Z(u)= R(u_R) + jI(u_I)\}$ **12** in FIG. 1. Notation introduced in FIG. 1 and 3 assumes that the user index $u=u_R=u_I$ for complex data symbols, and for real data symbols the user index u is used for counting the user pairs (u_R, u_I) of real and complex data symbols. These estimates are processed by the symbol decoder **30** and the frame processor **31** to recover estimates **32** of the transmitted user data words.

It should be obvious to anyone skilled in the communications art that this example implementation clearly defines the fundamental current CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

FIG. 4 real Walsh CDMA decoding is a representative implementation of the real Walsh CDMA decoding **27** in FIG. 3 and in equations (2). Inputs are the received estimates of the complex CDMA encoded chips $\{\hat{Z}(n)\}$ **33.** The PN scrambling code is stripped off from these chips **34** by changing the sign of each chip according to the numerical sign of the real and imaginary components of the complex conjugate of the PN code as per the decoding algorithms **8** in equations (2).

For complex data symbols **35** the real Walsh channelization coding is removed by a pulse compression operation consisting of

multiplying each received chip by the numerical sign of the corresponding Walsh chip for the user and summing the products over the N Walsh chips **36** to recover estimates $\{\hat{Z}(u)\}$ of the user complex data symbols {Z(u)}. The switch **35** selects the appropriate signal processing path for the complex and real data symbols.

For real data symbols **35** the next signal processing operation is the removal of the PN codes from the real and imaginary axes. This is followed by stripping off the real Walsh channelization coding by multiplying each received chip by the numerical sign of the corresponding Walsh chip for the user and summing the products over the N Walsh chips **36** to recover estimates $\{\hat{R}(u_R),\ \hat{I}(u_I)\}$ of the user real data symbols $\{R(u_R), I(u_I)\}$.

It should be obvious to anyone skilled in the communications art that this example implementation clearly defines the fundamental current CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

For cellular applications the transmitter description describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.

SUMMARY OF INVENTION

This invention is a new approach to the application of Walsh orthogonal codes for CDMA, which replaces the current real Walsh codes with the new complex Walsh codes and the hybrid complex Walsh codes disclosed in this invention. Real Walsh codes are used for current CDMA applications and will be used for all of the future CDMA systems. This invention of complex Walsh codes will provide the choice of using the new complex

Walsh codes or the real Walsh codes since the real Walsh codes are the real components of the complex Walsh codes. This means an application capable of using the complex Walsh codes can simply turn-off the complex axis components of the complex Walsh codes for real Walsh CDMA coding and decoding.

The complex Walsh codes of this invention are proven to be the natural development for the Walsh codes and therefore are the correct complex Walsh codes to within arbitrary factors that include scale and rotation, which are not relevant to performance. This natural development of the complex Walsh codes in the N-dimensional complex code space C^N extended the correspondences between the real Walsh codes and the Fourier codes in the N-dimensional real code space R^N, to correspondences between the complex Walsh codes and the discrete Fourier transform (DFT) codes in C^N.

The new 4-phase complex Walsh orthogonal CDMA codes provide fundamental performance improvements compared to the 2-phase real Walsh codes which include an increase in the carrier-to-noise ratio (CNR) for data symbol recovery in the receiver, lower correlation side-lobes under timing offsets both with and without PN spreading, lower levels of harmonic interference caused by non-linear amplification of multi-carrier CDMA signals, and reduced phase tracking jitter for code tracking to support both acquisition and synchronization. These potential performace improvements simply reflect the widely known principle that complex CDMA is better than real CDMA.

The new hybrid complex Walsh orthogonal CDMA codes increase the choices for the code length by allowing the combined use of complex Walsh and discrete Fourier transform complex orthogonal codes using a Kronecker construction, direct sum constructioin, as well as the possibility for more general functional combining.

BRIEF DESCRIPTION OF DRAWINGS AND PERFORMACE DATA

The above-mentioned and other features, objects, design algorithms, and performance advantages of the present invention

will become more apparent from the detailed description set forth below when taken in conjunction with the drawings and performance data wherein like reference characters and numerals denote like elements, and in which:

FIG. 1 is a representative CDMA transmitter signal processing implementation block diagram, with emphasis on the current real Walsh CDMA encoding which contains the signal processing elements addressed by this invention disclosure.

FIG. 2 is a representative CDMA encoding signal processing implementation diagram, with emphasis on the current real Walsh CDMA encoding which contains the signal processing elements addressed by this invention disclosure.

FIG.3 is a representative CDMA receiver signal processing implementation block diagram, with emphasis on the current real Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

FIG. 4 is a representative CDMA decoding signal processing implementation diagram, with emphasis on the current real Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

FIG.5 is a representative correlation plot of the correlation between the complex discrete Fourier transform (DFT) cosine and sine code component vectors and the real Fourier transform cosine and sine code component vectors.

FIG.6 is a representative CDMA encoding signal processing implementation diagram, with emphasis on the new complex Walsh CDMA encoding which contains the signal processing elements addressed by this invention disclosure

FIG.7 is a representative CDMA decoding signal processing implementation diagram, with emphasis on the new complex Walsh CDMA decoding which contains the signal processing elements addressed by this invention disclosure.

DISCLOSURE OF INVENTION

Real orthogonal CDMA code space R^N for Hadamard, Walsh, and Fourier codes: The new complex Walsh orthogonal CDMA codes are derived from the current real Walsh codes by starting with the correspondence of the current real Walsh codes with the discrete Fourier transform (DFT) basis vectors. Consider the real orthogonal CDMA code space R^N consisting of N-orthogonal real code vectors. Examples of code sets in R^N include the Hadamard, Walsh, and Fourier. The corresponding matrices of code vectors are designated as H, W, F respectively and as defined in equations (1) respectively consist of N-rows of N-chip code vectors. Hadamard codes in their reordered form known as Walsh codes are used in the current CDMA, in the proposals for the next generation G3 CDMA, and in the proposals for all future CDMA. Walsh codes reorder the Hadamard codes according to increasing sequency. These codes assumed +/-1 values. Sequency is the average rate of change of the sign of the codes and the reordering places the Walsh codes in correspondence to the DFT wherein sequency is in correspondence with frequency in the DFT.

It is important to note that the correspondence "sequency~frequency" only applies to the complex DFT matrix E consisting of the N-row vectors $\{E(u)= [E(u,0),\ldots,E(u,N-1)]$ wherein the elements of E are $E(u,n)=e^{j(2\pi\, un/N)}$, $u,n = 0,1,\ldots,N-1\}$. Historically it has not been applied to the Fourier basis F in R^N.

Equations (1) define the three sets H,W,F of real orthogonal codes in R^N with the understanding that the H and W are identical except for the ordering of the code vectors. Hadamard **37** and Walsh **38** orthogonal functions are basis vectors in R^N and are used as code vectors for orthogonal CDMA channelization coding. Hadamard **37** and Walsh **38** equations of definition are widely known with examples given in Reference [6]. Likewise, the Fourier **39** equations of definition are widely known within the engineering and scientific communities, wherein

N-chip real orthogonal CDMA codes **(3)**

37 Hadamard codes

H = Hadamard NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [H(u)] matrix of row vectors H(u)

= [H(u,n)] matrix of elements H(u,n)

H(u) = Hadamard code vector u

= [H(u,0), H(u,1), …, H(u,N-1)]

= 1xN row vector of chips H(u,0),….,H(u,N-1)

H(u,n) = Hadamard code u chip n

= +/+1 possible values

$$= (-1)\hat{}\ \left[\ \sum_{i=0}^{i=M-1} u_i n_i\ \right]$$

where $u = \sum_{i=0}^{i=M-1} u_i 2^i$ binary representation of u

$n = \sum_{i=0}^{i=M-1} n_i 2^i$ binary representation of n

38 Walsh codes

W = Walsh NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [W(u)] matrix of row vectors W(u)

= [W(u,n)] matrix of elements W(u,n)

W(u) =Walsh code vector u

= [W(u,0), W(u,1), …, W(u,N-1)]

W(u,n) = Walsh code u chip n

= +/+1 possible values

$$= (-1)\hat{}\ \left[u_{M-1} n_0 + \sum_{i=1}^{i=M-1} (u_{M-1-i} + u_{M-i}) n_i\ \right]$$

39 Fourier codes

F = Fourier NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [F(u)] matrix of row vectors F(u)

$$= \left[\frac{C}{S}\right]$$

C = N/2+1 x N matrix of row vectors C(u)

C(u) = Even code vectors for u=0,1,…,N/2

= [1, cos(2π u1/N),…,cos(2π u(N-1/N)]

S = N/2-1 x N matrix of row vectors S(u)

S(Δu) = Odd code vectors for u=N/2+Δu, Δu=1,2,…,N/2-1

= [sin(2π Δu 1/N),…,sin(2π Δu(N-1)/N)]

where F(u) = C(u) for u=0,1,…,N/2

= S(Δu) for Δu = u-N/2, u=N/2+1,…,N-1

the cosine C(u) and sine S(u) code vectors are the code vectors of the Fourier code matrix F.

Complex orthogonal CDMA code space C^N for DFT codes: The DFT orthogonal codes are a complex basis for the complex N-dimensional CDMA code space C^N and consist of the DFT harmonic code vectors arranged in increasing order of frequency. Equations (4) are the definition of the DFT code vectors. The DFT definition **40** is widely known within the engineering and scientific communities. Even and odd components of the DFT code vectors **41** are the real cosine code vectors {C(u)} and the imaginary sine code vectors (S(u)} where even and odd are referenced to the midpoint of the code vectors. These cosine and sine code vectors are the extended set 2N of the N Fourier cosine and sine code vectors.

N-chip DFT complex orthogonal CDMA codes **(4)**

40 DFT code vectors

E = DFT NxN orthogonal code matrix consisting of N rows of N chip code vectors

= [E(u)] matrix of row vectors E(u)

= [E(u,n)] matrix of elements E(u,n)

E(u) = DFT code vector u

= [E(u,0), E(u,1), …, E(u,N-1)]

= 1xN row vector of chips E(u,0),….,E(u,N-1)

E(u,n) = DFT code u chip n

$= e^{j2\pi\, un/N}$

= cos(2π un/N) + jsin(2π un/N)

= N possible values on the unit circle

41 Even and odd code vectors are the extended set of Fourier even and odd code vectors in **39** equations (3_)

$C(u)$ = Even code vectors for $u=0,1,\ldots,N-1$

$= [1, \cos(2\pi\ u\ 1/N),\ldots,\cos(2\pi\ u\ (N-1)/N)]$

$S(u)$ = Odd code vectors for $u=0,1,\ldots,N-1$

$= [0, \sin(2\pi\ u\ 1/N),\ldots,\sin(2\pi\ u\ (N-1)/N)]$

$E(u) = C(u) + j\ S(u)$ for $u=0,1,\ldots,N-1$

Complex orthogonal CDMA code space C^N for complex Walsh codes: Step 1 in the derivation of the complex Walsh codes in this invention establishes the correspondence of the even and odd Walsh codes with the even and odd Fourier codes. Even and odd for these codes are with respect to the midpoint of the row vectors similar to the definition for the DFT vector codes **41** in equations (4). Equations (5) identify the even and odd Walsh codes in the W basis in R^N. These even and odd Walsh codes can be placed in

Even and odd Walsh codes in R^N **(5)**

$W_e(u)$ = Even Walsh code vector

$= W(2u)$ for $u=0,1,\ldots,N/2-1$

$W_o(u)$ = Odd Walsh code vectors

$= W(2u-1)$ for $u=1,\ldots,N/2$

direct correspondence with the Fourier code vectors **39** in equations (3) using the DFT equations (4). This correspondence is defined in equations (6) where the correspondence operator "~" represents the even and odd correspondence between the Walsh and Fourier codes, and additionally represents the sequency~frequency corresponcence.

Correspondence between Walsh and Fourier codes **(6)**

$W(0) \sim C(0)$

$W_e(u) \sim C(u)$ for $u=1,\ldots,N/2-1$

$W_o(u) \sim S(u)$ for $u=1,\ldots,N/2-1$

$W(N-1) \sim C(N/2)$

Step 2 derives the set of N complex DFT vector codes in C^N from the set of N real Fourier vector codes in R^N. This means that the set of 2N cosine and sine code vectors in **41** in equations (4) for the DFT codes in C^N will be derived from the set of N cosine and sine code vectors in **39** in equations (3) for the Fourier codes in R^N. The first N/2+1 code vectors of the DFT basis can be written in terms of the Fourier code vectors in equations (7).

DFT code vectors 0,1,…,N/2 derived from Fourier **(7)**

42 Fourier code vectors from **39** in equations (3) are

$C(u)$ = Even code vectors for $u=0,1,\ldots,N/2$

$= [1, \cos(2\pi\, u1/N),\ldots,\cos(2\pi\, u(N-1/N)]'$

$S(u)$ = Odd code vectors for $u=1,2,\ldots,N/2-1$

$= [\sin(2\pi\, u1/N),\ldots,\sin(2\pi\, u(N-1)/N)]$

43 DFT code vectors in **41** of equations (4) are written as functions of the Fourier code vectors

$E(u)$ = DFT complex code vectors for $u=0,1,\ldots,N/2$

$= C(0)$

$= C(u)+jS(u)$ for $u=1,\ldots,N/2-1$

$= C(N/2)$ for $u=N/2$

The remaining set of N/2+1,…,N-1 DFT code vectors in C^N can be derived from the original set of Fourier code vectors by a correlation which establishes the mapping of the DFT codes onto the Fourier codes. We derive this mapping by correlating the real and imaginary components of the DFT code vectors with the corresponding even and odd components of the Fourier code vectors. The correlation operation is defined in equations (8)

Correlation of DFT and Fourier code vectors **(8)**

$Corr(even) = C\ Real\{E'\}$

= Correlation matrix

= Matrix product of C and the real part of E transpose

$Corr(odd) = S\ Imag\{E'\}$

= Correlation matrix

= Matrix product of S and the imaginary part of E transpose

and the results of the correlation calculations are plotted in FIG.5 for N=32 for the real cosine and the odd sine Fourier code vectors. Plotted are the correlation of the 2N DFT cosine and sine codes against the N Fourier cosine and sine codes which range from -15 to +16 where the negative indices of the codes represent a negative correlation value. The plotted curves are the correlation peaks. These correlation curves in FIG. 5 prove that the remaining N/2+1,…,N-1 code vectors of the DFT are derived from the Fourier code vectors by equations (9)

DFT code vectors N/2+1,…, N-1 derived from Fourier **(9)**

$$E(u) = C(N/2 - \Delta u) - jS(N/2 - \Delta u)$$

for $u = N/2 + \Delta u$

$\Delta u = 1, \ldots, N/2-1$

This construction of the remaining DFT basis in equations (9) is an application of the DFT spectral foldover property which observes the DFT harmonic vectors for frequencies $fNT=N/2+\Delta i$ above the Nyquist sampling rate $fNT=N/2$ simply foldover such that the DFT harmonic vector for $fNT = N/2+\Delta i$ is the DFT basis vector for $fNT = N/2-\Delta i$ to within a fixed sign and fixed phase angle of rotation.

Step 3 derives the complex Walsh code vectors from the real Walsh code vectors by using the DFT derivation in equations (7) and (9), by using the correspondences between the real Walsh and Fourier in equations (6), and by using the fundamental correspondence between the complex Walsh and the complex DFT given in equation (10). We start by constructing the complex

Correspondence between complex Walsh and DFT **(10)**

$\tilde{W} \sim E$ NxN complex DFT orthogonal code matrix

where $\tilde{W}$ = NxN complex Walsh orthogonal code matrix

= N rows of N chip code vectors

$= [\ \tilde{W}(u)\ ]$ matrix of row vectors $\tilde{W}(u)$

$= [\ \tilde{W}(u,n)\ ]$ matrix of elements $\tilde{W}(u,n)$

$\tilde{W}(u)$ = Complex Walsh code vector u

$= [\ \tilde{W}(u,0),\ \tilde{W}(u,1),\ \ldots,\ \tilde{W}(u,N-1)\ ]$

$\tilde{W}$ = +/-1 +/- j possible value

Walsh dc code vector $\tilde{W}(0)$. We use equation E(0)=C(0) in **43** in equations (7), the correspondence in equations (6), and observe that the dc complex Walsh vector has both real and imaginary components in the $\tilde{W}$ domain, to derive the dc complex Walsh code vector:

$$\tilde{W}(0) = W(0) + jW(0) \qquad \text{for } u=0 \qquad \textbf{(11)}$$

For complex Walsh code vectors $\tilde{W}(u)$, u=1,2,…,N/2-1, we apply the correspondences in equations (10) between the complex Walsh and DFT bases, to the DFT equations **43** in equations (7):

$$\begin{aligned} \tilde{W}(u) &= W_e(u) + jW_o(u) && \text{for } u=1,2,\ldots,N/2-1 \qquad \textbf{(12)} \\ &= W(2u) + jW(2u-1) && \text{for } u=1,2,\ldots,N/2-1 \end{aligned}$$

For complex Walsh code vector $\tilde{W}(N/2)$ we use the equation E(N/2)=C(N/2) **43** in equations (7) and the same rationale used to derive equation (11), to yield the equation for $\tilde{W}(N/2)$.

$$\tilde{W}(N/2) = W(N-1) + jW(N-1) \qquad \text{for } u=N/2 \qquad \textbf{(13)}$$

For complex Walsh code vectors $\tilde{W}(N/2+\Delta u)$, Δu=1,2,…,N/2-1 we apply the correspondences between the complex Walsh and DFT bases to the spectral foldover equation $E(N/2+\Delta u)=C(N/2-\Delta u)-jS(N/2-\Delta u)$ in equations (9) with the changes in indexing required to account for the W indexing in equations (5). The equations are

$$\begin{aligned} \tilde{W}(N/2+\Delta u) &= W(N-1-\Delta eu)+W(N-1-\Delta ou) && \text{for } u=N/2+1,\ldots,N-1 \ \textbf{(14)} \\ &= W(N-1-2\Delta u) + jW(N-2\Delta u) && \text{for } u=N/2+1,\ldots,N-1 \end{aligned}$$

using the notation $\Delta ei=2\Delta i$, $\Delta oi=2\Delta i-1$. These complex Walsh code vectors in equations (11), (12), (13), (14) are the equations of definition for the complex Walsh code vectors.

An equivalent way to derive the complex Walsh code vectors in C^N from the real Walsh basis in R^{2N} is to use a sampling technique which is a known method for deriving a complex basis in C^N from a real basis in R^N.

Transmitter equations (15) describe a representative complex Walsh CDMA encoding for the transmitter in FIG. 1. It is assumed that there are N complex Walsh code vectors $\widetilde{W}(u)$ **44** each of length N chips similar to the definitions for the real Walsh code vectors **1** in equations (1). The code vector is presented by a 1xN N-chip row vector $\widetilde{W}(u) = [\widetilde{W}(u,0), \ldots, \widetilde{W}(u,N\text{-}1)]$ where $\widetilde{W}(u,n)$ is chip n of code u. The code vectors are the row vectors of the complex Walsh matrix $\widetilde{W}$. Walsh code chip n of code vector u has the possible values $\widetilde{W}(u,n) = +/-1 +/-j$. Each user is assigned a unique Walsh code which allows the code vectors to be designated by the user symbols u=0,1,…,N-1 for N complex Walsh codes. The complex Walsh code vectors $\widetilde{W}(u)$ derived in equations (11),(12),(13),(14) are summarized **44** in terms of their real and imaginary component code vectors $\widetilde{W}(u) = W_R(u) + jW_I(u)$ where $W_R(u)$ and $W_I(u)$ are respectively the real and imaginary component code vectors. As per the derivation of $\widetilde{W}(u)$ the sets of real axis code vectors $\{W_R(u)\}$ and the imaginary axis code vectors $\{W_I(u)\}$ both consist of the real Walsh code vectors in R^N with the ordering modified to ensure that the definition of the complex Walsh vectors satisfies equations (11), (12), (13), (14).

Complex Walsh CDMA encoding for transmitter (15)

44 Complex Walsh codes use the definitions for the real Walsh codes in **1** equations (1) and the definitions of the complex Walsh codes in equations (11), (12), (13), (14). We find

$\widetilde{W}$ = complex Walsh NxN orthogonal code matrix

consisting of N rows of N chip code vectors

$= [\ \widetilde{W}(u)\ ]$ matrix of row vectors $\widetilde{W}(u)$

$= [\widetilde{W}(u,n)\ ]$ matrix of elements $\widetilde{W}(u,n)$

$\widetilde{W}(u)$ = complex Walsh code vector u

$= W_R(u) + jW_I(u)$ for $u=0,1,\ldots,N-1$

where

$W_R(u) = \mathrm{Real}\{\ \widetilde{W}(u)\ \}$

$= W(0)$ for $u=0$

$= W(2u)$ for $u=1,2,\ldots,N/2-1$

$= W(N-1)$ for $u=N/2$

$= W(2N-2u-1)$ for $u=N/2+1,\ldots,N-1$

$W_I(u) = \mathrm{Imag}\{\ \widetilde{W}(u)\ \}$

$= W(0)$ for $u=0$

$= W(2u-1)$ for $u=1,2,\ldots,N/2-1$

$= W(N-1)$ for $u=N/2$

$= W(2N-2u)$ for $u=N/2+1,\ldots,N-1$

$\widetilde{W}(u,n)$ = complex Walsh code u chip n

$= +/-1 +/-j$ possible values

45 Data symbols

$Z(u)$ = Complex data symbol for user u

$= R(u) + jI(u)$

46 Complex Walsh encoded data

$$Z(u,n) = Z(u)\ \widetilde{W}(u,n)$$

$$= Z(u)[\mathrm{sgn}\{W_R(u,n)\} + j\mathrm{sgn}\{W_I(u,n)\}]$$

$$= [R(u)\mathrm{sgn}\{W_R(u,n)\} - I(u)\mathrm{sgn}\{W_I(u,n)\}]$$

$$+ j[R(u)\mathrm{sgn}\{W_I(u,n) + I(u,n)\mathrm{sgn}\{W_R(u,n)\}]$$

47 PN scrambling

$P_R(n)$ = Chip n of the PN code for the real axis

$P_I(n)$ = Chip n of the PN code for the imaginary axis

$Z(n)$ = PN scrambled complex Walsh encoded data chips after summing over the users

$$= \sum_u Z(u,n)[\mathrm{P_R(n)} + \mathrm{j\,P_I(n)}]$$

$$= \sum_u Z(u,n)[\mathrm{sgn}\{\mathrm{P_R(n)}\} + \mathrm{j}\,\mathrm{sgn}\{\mathrm{P_I(n)}\}]$$

= Complex Walsh CDMA encoded chips

User data symbols **45** are the set of complex symbols $\{Z(u),\ u=0,1\ldots,N-1\}$. These data symbols are encoded by the Walsh CDMA codes **46.** Each of the user symbols Z(u) is assigned a unique complex Walsh code $\widetilde{W}(u)=W_R(u)+jW_I(u)$. Complex Walsh encoding of each user data symbol generates an N-chip sequence with each chip in the sequence consisting of the user data symbol with the complex sign of the corresponding complex Walsh code chip, which means each encoded chip = [Data symbol Z(u)] x [Sign of $W_R(u)$ + j sign of $W_I(u)$].

The complex Walsh encoded data symbols are summed and encoded with PN scrambling codes **47.** These PN codes are defined **4** in equations (1) as a complex PN for each chip n, equal to $[P_R(u) + j\ P_I(u)]$ where $P_R(u)$ and $P_I(u)$ are the respective PN scrambling codes for the real and imaginary axes. Encoding with the complex PN is the same as given **4** in equations (1) for complex data symbols. Each complex Walsh encoded data chip Z(u,n) **46** is summed over the set of users $u=0,1,\ldots,N-1$ and complex PN encoded to yield the complex Walsh CDMA chips Z(n) = $\sum_u Z(u,n)\ [P_R(n)\ + j\ P_I(n)]$ **47.**

Although not considered in this example, it is possible to use combinations of both complex and real data symbols similar to the approach for real Walsh CDMA encoding in equations (1) since the complex Walsh code vectors are the real Walsh code vectors along the real axis and a reordering of the real Walsh code vectors along the imaginary axis.

Receiver equations (16) describe a representative complex Walsh CDMA decoding for the receiver in FIG. 3. The receiver front end **48** provides estimates $\{\hat{Z}(n)\}$ of the transmitted complex Walsh CDMA encoded chips {Z(n)} for the complex data symbols {Z(u)}. Orthogonality property **49** is expressed as a

matrix product of the complex Walsh code chips or equivalently as a matrix produce of the complex Walsh code chip numerical signs of the real and imaginary components. The 2-phase PN codes **50** have the useful decoding property that the square of each code chip is unity which is equivalent to observing that the square of each code chip numerical sign is unity. Decoding algorithms **51** perform the inverse of the signal processing for the encoding in equations (15) to recover estimates $\{\hat{Z}(u)\}$ of the transmitter user symbols {Z(n)} for the complex data symbols {Z(u)}.

Complex Walsh CDMA decoding for receiver **(16)**

48 Receiver front end in FIG. 3 provides estimates

$\{\hat{Z}(n)\}$ **28** of the encoded transmitter chip symbols

{Z(n)} **47** in equations (15).

49 Orthogoality property of complex Walsh NxN matrix $\widetilde{W}$

$$\sum_n \widetilde{W}(\hat{u},n)\widetilde{W}^*(n,u) =$$

$$\sum_n [\operatorname{sgn}\{W_R(\hat{u},n)\}+j\operatorname{sgn}\{W_I(\hat{u},n)\}][\operatorname{sgn}\{W_R(n,u)-j\operatorname{sgn}\{W_I(n,u)\}]$$

$$= 2N\ \delta(\hat{u}, u)$$

where $\delta(\hat{u}, u)$ = Delta function of $\hat{u}$ and u

= 1 for $\hat{u} = u$

= 0 otherwise

50 PN decoding property

P(n)P(n) = sgn{P(n)} sgn{P(n)}

= 1

51 Decoding algorithm

$$\hat{Z}(u) = 2^{-1}N^{-1}\sum_n \hat{Z}(n)[\operatorname{sgn}\{P_R(n)\}-j\operatorname{sgn}\{P_I(n)\}][\operatorname{sgn}\{W_R(n,u)\}-j\operatorname{sgn}\{W_I(n,u)\}]$$

= Receiver estimate of the transmitted data symbol Z(u) **45** in equations (15)

Although not considered in this example, it is possible to use combinations of both complex and real data symbols similar to the approach for real Walsh CDMA decoding in FIG. 4 since the complex Walsh code vectors are the real Walsh code vectors along

the real axis and a reordering of the real Walsh code vectors along the imaginary axis.

FIG. 6 complex Walsh CDMA encoding is a representative implementation of the complex Walsh CDMA encoding which will replace the current real Walsh encoding **13** in FIG. 1, and defined in equations (15). Inputs are the user data symbols $\{Z(u)\}$ **52.** Encoding of each user by the corresponding complex Walsh code is described in **53** by the implementation of transferring the sign $+/-1+/-j$ of each complex Walsh code chip to the user data symbol followed by a 1-to-N expander $1\uparrow N$ of each data symbol into an N chip sequence using the sign transfer of the complex Walsh chips.

The sign-expander operation **53** generates the N-chip sequence $Z(u,n)=Z(u)[\mathrm{sgn}\{W_R(u,n)\}+j\mathrm{sgn}\{W_I(u,n)\}]=Z(u)[W_R(u,n)+jW_I(u,n)]$ for $n=0,1,\ldots,N-1$ for each user $u=0,1,\ldots,N-1$. This complex Walsh encoding serves to spread each user data symbol into an orthogonally encoded chip sequence which is spread over the CDMA communications frequency band. The complex Walsh encoded chip sequences for each of the user data symbols are summed over the users **54** followed by PN encoding with the scrambling sequence $[P_R(n)-jP_I(n)]$ **55.** PN encoding is implemented by transferring the sign of each PN chip to the summed chip of the Walsh encoded data symbols. Output is the stream of complex CDMA encoded chips $\{Z(n)\}$ **56.**

Although not considered in this example, it is possible to use combinations of both complex and real data symbols similar to the approach for real Walsh CDMA encoding in FIG. 2 since the complex Walsh code vectors are the real Walsh code vectors along the real axis and a reordering of the real Walsh code vectors along the imaginary axis.

It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 6 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example

is representative of the other possible signal processing approaches.

FIG. 7 complex Walsh CDMA decoding is a representative implementation of complex Walsh CDMA decoding which will replace the current real Walsh decoding **27** in FIG. 3, and defined in equations (15). Inputs are the received estimates of the complex CDMA encoded chips $\{\hat{Z}(n)\}$ **57.** The PN scrambling code is stripped off from these chips **58** by changing the sign of each chip according to the numerical sign of the real and imaginary components of the complex conjugate of the PN code as per the decoding algorithms **50** in equations (16).

The complex Walsh channelization coding is removed by a pulse compression operation consisting of multiplying each received chip by the numerical sign of the corresponding complex Walsh chip for the user and summing the products over the N Walsh chips **59** to recover estimates $\{\hat{Z}(u)\}$ of the user complex data symbols {Z(u)}.

Although not considered in this example, it is possible to use combinations of both complex and real data symbols similar to the approach for real Walsh CDMA decoding in FIG. 4 since the complex Walsh code vectors are the real Walsh code vectors along the real axis and a reordering of the real Walsh code vectors along the imaginary axis.

It should be obvious to anyone skilled in the communications art that this example implementation in FIG. 6 clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

For cellular applications the transmitter description describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.

Complex orthogonal CDMA code space C^N for hybrid complex Walsh codes: The power of 2 code lengths N=2^M where M is an integer, for complex Walsh can be modified to allow the code length N to be a product of powers of primes **60** in equations (17) or a sum of powers of primes **61** in equations (17), at the implementation cost of introducing multiply operations into the CDMA encoding and decoding. In the previous disclosure of this invention we used N equal to a power of 2 which means N=2^m corresponding to $p_0 = 2$ and $M=m_0$. This restriction was made for convenience in explaining the construction of the complex Walsh and is not required since it is well known that Hadamard matrices exist for non-integer powers of 2 and, therefore, complex Walsh matrices exist for non-integer powers of 2.

Length N of hybrid complex Walsh orthogonal codes **(17)**

60 Kronecker product code construction

$$N = \prod_k p_k^{m_k}$$
$$= \prod_k N_k$$

where

P_k = prime number indexed by k starting with k=0

m_k = order of the prime number p_k

N_k = Length of code for the prime p_k

$= p_k^{m_k}$

61 Direct sum code construction

$$N = \sum_k p_k^{m_k}$$
$$= \sum_k N_k$$

Add-only arithemetic operations are required for encoding and decoding both real Walsh and complex Walsh CDMA codes since the real Walsh values are +/-1 and the complex Walsh values are +/-1 +/-j which means the only operations are sign transfer and adds plus subtracts or add-only. Multiply operations are more

complex to implement than add operations. However, the advantages of having greater flexibility in choosing the orthogonal CDMA code lengths N using equations (17) can offset the expense of multiply operations for particular applications. Accordingly, this invention includes the concept of hybrid complex Walsh orthogonal CDMA codes with the flexibility to meet these needs. This extended class of complex Walsh codes are hybrid in that their construction supplements the complex Walsh codes with the use of Hadamard (or real Walsh), DFT, and other orthogonal codes.

Hybrid complex Walsh orthogonal CDMA codes can be constructed as demonstrated in **64** and **65** in equations (18) for the Kronecker product, and in **66** in equations (18) for the direct sum. Code matrices considered **62** in equations (18) for the construction of the hybrid complex Walsh are the DFT E and Hadamard H, in addition to the complex Walsh $\widetilde{W}$. The algorithms and examples for the construction start with the definitions **63** of the NxN orthogonal code matrices $\widetilde{W}_N$, E_N, H_N for $\widetilde{W}$, E, H respectively, examples for low orders N=2,4, and the equivalence of E_4 and $\widetilde{W}_4$ after the $\widetilde{W}_4$ is rotated through the angle -90 degrees and rescaled. The CDMA current and developing standards use the prime 2 which generates a code length $N=2^M$ where M=integer. For applications requiring greater flexibility in code length N, additional primes can used using the Kronecker construction. We illustrate this in **65** with the addition of prime=3. The use of prime=3 in addition to the prime=2 in the range of N=8 to 64 is observed to increase the number of N choices from 4 to 9 at a modest cost penality of using multiples of the angle increment 30 degrees for prime=3 in addition to the angle increment 90 degrees for prime=2. As noted in **65** there are several choices in the ordering of the Kronecker product construction and 2 of these choices are used in the construction.

Direct sum construction provides greater flexibility in the choice of N without necessarily introducing a multiply penality. However, the addition of the zero matrix in the construction is generally not desirable for CDMA communications.

Construction of hybrid complex Walsh orthogonal codes (18)

62 Code matrices

$\widetilde{W}_N$ = NxN complex Walsh orthogonal code matrix

E_N = NxN DFT orthogonal code matrix

H_N = NxN Hadamard orthogonal code matrix

63 Low-order code definitions and equivalences

2x2 $$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= E_2$$

$$= (e^{-j\pi/4}/\sqrt{2}) * \widetilde{W}_2$$

3x3 $$E_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j2\pi/3} & e^{j2\pi 2/3} \\ 1 & e^{j2\pi 2/3} & e^{j2\pi/3} \end{bmatrix}$$

4x4 $$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\widetilde{W}_4 = \begin{bmatrix} 1+j & 1+j & 1+j & 1+j \\ 1+j & -1+j & -1-j & 1-j \\ 1+j & -1-j & 1+j & -1-j \\ 1+j & 1-j & -1-j & -1+j \end{bmatrix}$$

$$E_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ & & & \\ & & & \end{bmatrix}$$

$$\begin{matrix} 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{matrix}$$

$$= \quad (e^{-j\pi/4} / \sqrt{2}) \ \widetilde{W}_4$$

64 Kronecker product construction for $N = \prod_k N_k$

Code matrix C_N = NxN hybrid orthogonal CDMA code matrix

Kronecker product construction of C_N

$$C_N = C_0 \prod_{k>0} \otimes C_{N_k}$$

Kronecker product definition

A = $N_a x N_a$ orthogonal code matrix

B = $N_b x N_b$ orthogonal code matrix

$A \otimes B$ = Kronecker product of matrix A and matrix B

= $N_a N_b$ x $N_a N_b$ orthogonal code matrix consisting of the elements $[a_{ik}]$ of matrix A multiplied by the matrix B

= $[\ a_{ik} B\ ]$

65 Kronecker product construction examples for primes p=2,3 and the range of sizes $8 \le N \le 64$

8x8 $\quad C_8 = \widetilde{W}_8$

12x12 $\quad C_{12} = \widetilde{W}_4 \otimes E_3$

$\quad C_{12} = E_3 \otimes \widetilde{W}_4$

16x16 $\quad C_{16} = \widetilde{W}_{16}$

18x18 $\quad C_{18} = \widetilde{W}_2 \otimes E_3 \otimes E_3$

$\quad C_{18} = E_3 \otimes E_3 \otimes \widetilde{W}_2$

24x24 $\quad C_{24} = \widetilde{W}_8 \otimes E_3$

$\quad C_{24} = E_3 \otimes \widetilde{W}_8$

32x32 $\quad C_{32} = \widetilde{W}_{32}$

36x36 $\quad C_{36} = \widetilde{W}_4 \otimes \widetilde{W}_3 \otimes \widetilde{W}_3$

$$C_{36} = \widetilde{W}_3 \otimes \widetilde{W}_3 \otimes \widetilde{W}_4$$

48x48 $$C_{48} = \widetilde{W}_{16} \otimes \widetilde{W}_3$$

$$C_{48} = \widetilde{W}_3 \otimes \widetilde{W}_{16}$$

64x64 $$C_{64} = \widetilde{W}_{64}$$

66 Direct sum construction for $N=\sum_k N_k$

Code matrix C_N = NxN hybrid orthogonal CDMA code matrix

Direct sum construction of C_N

$$C_N = C_0 \prod_{k>0} \oplus C_{N_k}$$

Direct sum definition

A = $N_a x N_a$ orthogonal code matrix

B = $N_b x N_b$orthogonal code matrix

$A \oplus B$ = Direct sum of matrix A and matrix B

= N_a+N_b x N_a+N_b orthogonal code matrix

$$= \left[\begin{array}{c|c} A & O_{N_a x N_b} \\ \hline O_{N_b x N_a} & B \end{array}\right]$$

where $O_{N_1 x N_2}$ = $N_1 x N_1$ zero matrix

It should be obvious to anyone skilled in the communications art that this example implementation of the hybrid complex Walsh in equations (18) clearly defines the fundamental CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches. For example, the Kronecker matrices E_N and H_N can be replaced by functionals.

For cellular applications the transmitter description which includes equations (18) describes the transmission signal processing applicable to this invention for both the hub and user terminals, and the receiver corresponding to the decoding of equations (18) describes the corresponding receiving signal processing for the hub and user terminals for applicability to this invention.

Computationally efficient encoding and decoding of complex Walsh CDMA codes and hybrid complex Walsh CDMA codes: It is well known that fast and efficient encoding and decoding algorithms exist for the real Walsh CDMA codes. These are documented in reference [6]. It is obvious that with suitable modifications these algorithms can be used to develop fast and efficient encoding and decoding algorithms for the complex Walsh CDMA codes since these complex codes have real and imaginary code vectors which are from the same set of real Walsh CDMA codes.

It is well known that the Kronecker construction involving DFT and real Walsh orthogonal code vectors have efficient encoding and decoding algorithms. It is obvious that with suitable modifications these algorithms can be used to develop fast and efficient encoding and decoding algorithms for the Kronecker products of DFT and complex Walsh CDMA codes since these complex Walsh codes have real and imaginary code vectors which are from the same set of real Walsh CDMA codes.

Preferred embodiments in the previous description is provided to enable any person skilled in the art to make or use the present invention. The various modifications to these embodiments will be readily apparent to those skilled in the art, and the generic principles defined herein may be applied to other embodiments without the use of the inventive faculty. Thus, the present invention is not intended to be limited to the embodiments shown herein but is not to be accorded the wider scope consistent with the principles and novel features disclosed herein.

WHAT IS CLAIMED IS:

1. A means for the design of new complex Walsh orthogonal CDMA encoding and decoding over a frequency band with properties

provide a complex Walsh orthogonal code with the real component equal to the real Walsh orthogonal code

provide a complex Walsh orthogonal code with the imaginary component equal to a reordering of the real Walsh orthogonal code, which makes the complex Walsh orthogonal code the correct complex version of the real Walsh orthogonal code to within arbitrary angle rotations and scale factors

provide a complex Walsh orthogonal code which is in correspondence with the discrete Fourier transform (DFT) complex orthogonal codes wherein the correspondence is twofold: the sequency of the complex Walsh orthogonal codes is the average rate of rotation of the complex Walsh codes and corresponds to the frequency of the DFT codes with sequency as well as frequency increasing with the code numbering, and the second correspondence is between the even and odd complex Walsh code vectors and the cosine and sine DFT code vectors respectively

provide a complex Walsh orthogonal code which has the sign values +/-1 +/-j for the real and imaginary axes

provide a complex Walsh orthogonal code which has a fast decoding algorithm

provide a hybrid complex Walsh orthogonal code which can be constructed for a wide range of code lengths by combining the complex Walsh codes with DFT complex orthogonal codes

2. A means for the design of new complex Walsh orthogonal CDMA codes with the properties

provide complex Walsh orthogonal CDMA codes which reduce to the real Walsh orthogonal CDMA codes upon removal of the complex code components

provide complex Walsh orthogonal CDMA codes which reduce to the real Walsh orthogonal CDMA codes upon removal of the real code components

provide a means for the computational efficient encoding and decoding of the complex Walsh orthogonal CDMA codes

3. A means for the design of new complex Walsh orthogonal CDMA codes with the properties

provide the correct generalization of the real Walsh orthogonal CDMA codes to the complex Walsh orthogonal CDMA codes

provide a computationally efficient means to encode and decode the complex Walsh orthogonal CDMA codes

provide a means to extend the complex Walsh orthogonal CDMA codes to include the complex discrete Fourier transform (DFT) codes to allow greater flexibility in the choices for the code lengths

4. A means for the design of hybrid complex Walsh orthogonal CDMA codes with the properties

provide a means to provide greater flexibility in the selection of the code length by combining the complex Walsh orthogonal CDMA codes with the complex DFT orthogonal CDMA codes

provide a Kronecker product means to combine the complex Walsh orthogonal CDMA codes with the complex DFT orthogonal CDMA codes

provide a direct sum means to combine the complex Walsh orthogonal CDMA codes with complex DFT orthogonal CDMA codes as well as other complex Walsh orthogonal CDMA codes

provide a functionality means to combine the complex Walsh orthogonal CDMA codes with the complex DFT orthogonal CDMA codes

4. A means for the design of 4-phase Walsh orthogonal CDMA codes with the properties

provide 4-phase Walsh orthogonal CDMA codes which can be reduced to the 2-phase real Walsh orthogonal CDMA codes

provide 4-phase Walsh orthogonal CDMA codes which are the correct generalization of the 2-phase real Walsh orthogonal CDMA codes to 4-phases

provide hybrid Walsh orthogonal CDMA codes by combining the 4-phase Walsh orthogonal codes with the N-phase DFT codes with greater flexibility in the choice of the code length

5. A means for the design of 4-phase Walsh orthogonal CDMA codes with the properties

provide 4-phase Walsh orthogonal CDMA codes in the code space C^N which include the 2-phase real Walsh orthogonal CDMA codes in R^N

provide 4-phase Walsh orthogonal CDMA codes which have computationally efficient encoding and decoding implementation algorithms

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